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Increasing Average Period Lengths by Switching of Robust Chaos Maps in Finite Precision

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Abstract. Grebogi, Ott and Yorke (Phys. Rev. A 38(7), 1988) have investigated the effect of finite precision on average period length of chaotic maps. They showed that the average length of periodic orbits (T) of a dynamical system scales as a function of computer precision (ε) and the correlation dimension (d) of the chaotic attractor: $T \sim \varepsilon^{-d/2}$. In this work, we are concerned with increasing the average period length which is desirable for chaotic cryptography applications. Our experiments reveal that random and chaotic switching of deterministic chaotic dynamical systems yield higher average length of periodic orbits as compared to simple sequential switching or absence of switching. To illustrate the application of switching, a novel generalization of the Logistic map that exhibits Robust Chaos (absence of attracting periodic orbits) is first introduced. We then propose a pseudo-random number generator based on chaotic switching between Robust Chaos maps which is found to successfully pass stringent statistical tests of randomness.

1 Introduction

Grebogi, Ott and Yorke (Phys. Rev. A 38(7), 1988) were one of the first to study in detail the effect of finite precision on the expected length of periodic orbits and their distribution for chaotic maps. On a digital computer, since there are only a finite number of states owing to limited precision, all autonomous chaotic maps (and flows) when simulated would have to eventually settle down to periodic orbits for all initial conditions (Figure 1(a), T is defined as the period length). Thus Chaos which is characterized by the existence of wandering orbits which have infinite period length is impossible on a digital computer.

This fact is often under appreciated. Chaotic cryptographic applications appeal to the fact that Chaos has inherently good mixing properties which are suitable for confusing and diffusing the message. However, owing to limited precision, this is not strictly true since there would be only a finite number of periodic orbits and no wandering orbits. This probably explains why many chaotic cryptographic algorithms have been eventually broken though they initially promised to exhibit strong theoretical security.

The question that naturally arises is: What is the average length of the periodic orbits of the dynamical system implemented on a finite precision computer? Grebogi's work [1] showed that the average length of periodic orbits (T) of a dynamical system scales as a function of computer precision (ε) and the correlation dimension (d, it is defined as the exponent of the power-law dependence of the correlation integral and is a measure of the strangeness of the

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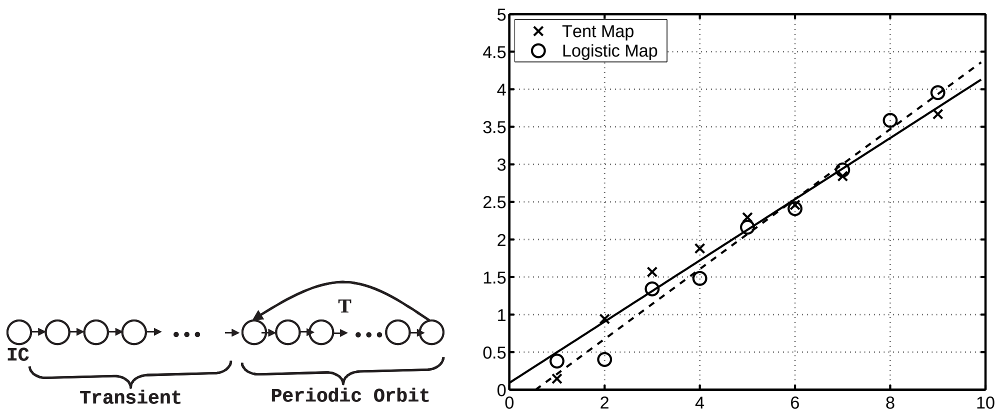

Fig. 1. (a) Left: All initial conditions of a dynamical system when iterated on a finite precision computer end up in a periodic orbit. (b) Right: Average period length T vs. $(1/\varepsilon)$ in logarithmic scale for the Tent map ($T = 10^{0.0904}\varepsilon^{-0.4075}$) and the Logistic map ($T = 10^{-0.2576}\varepsilon^{-0.4658}$). This agrees quite well with the relationship derived by Grebogi [1].

attractor [2]) of the chaotic attractor: $T \sim \varepsilon^{-d/2}$. Figure 1(b) shows the plot of the average period length T with respect to $(1/\varepsilon)$ (logarithmic scale) for the Tent map ($x \mapsto 2x, 0 \leq x < 0.5;\ \ x \mapsto 2 - 2x, 0.5 \leq x \leq 1$) and the Logistic map ($x \mapsto 4x(1-x)$). In both cases, $x \in [0,1]$. As it can be seen, the relationship $T \sim \varepsilon^{-d/2}$ is empirically well matched in both cases.

Subsequently, there have been only few studies on the effect of computer precision on the average period length. Wang et. al. [3] discovered that a single dominant periodic trajectory is realized with major probability. They also studied coupled maps and showed that these exhibit larger period lengths which can be useful in chaotic cryptography. The other notable work with regards to the effect of computer precision to chaotic cryptography is that of Li [4] where they performed quantitative analysis of the degradation of digitized chaos and proposed a new series of dynamical indicators for 1D piecewise linear chaotic maps.

The question that we are interested in is – How can we increase the average period length T while still using the same precision? This paper is organized as follows. In Section 2, the effect of switching between chaotic maps on the average period length and how different switching strategies increase the average period length is discussed. In Section 3, a pseudo-random number generator is built to demonstrate the applications of switching of chaotic maps. To this end, a generalization of the Logistic map that exhibits Robust Chaos is proposed. Robust Chaos is theoretically characterized by absence of attracting periodic orbits. This property is highly desirable for chaotic cryptographic applications. The pseudo-random number generator switches between these family of maps (still under finite precision). In Section 4, randomness evaluation of the pseudo-random number generator to demonstrate its merit is performed and we conclude in Section 5.

2 Switching of chaotic maps and its effect on average period length

We consider switching between chaotic maps and study the effect of different switching strategies on the average period length. What we mean by switching of maps is as follows. We start with an initial condition (typically chosen at random) and iterate with the first dynamical system (say the Tent map). For the sequential switching strategy, after one iteration of the first dynamical system, we iterate the second dynamical system (say the Logistic map). We then iterate the first dynamical system again and so on and so forth. For a chaotic or random switching strategy, we randomly or chaotically choose the dynamical system to iterate at every iteration. We then determine the period length and repeat this for a number of initial conditions and take their average. We do this for several precision values.

We chose the Tent map, the Logistic map and the Skewed Tent map ($x \mapsto \frac{x}{p}, 0 \leq x < p;\quad x \mapsto \frac{1-x}{1-p}, p \leq x \leq 1$) with values of $p = 0.1$ and $p = 0.2$. The switching strategies we used were sequential, chaotic (Logistic map) and random (Mersenne Twister [5] and RAND(.)). The experiments were performed on a computer with the following specifications – Pentium® 4, CPU 3.06 GHz, 480 M bytes RAM (C program). For smaller ε, we averaged the period lengths over 1000 randomly chosen initial conditions. The results are shown in Figure 2.

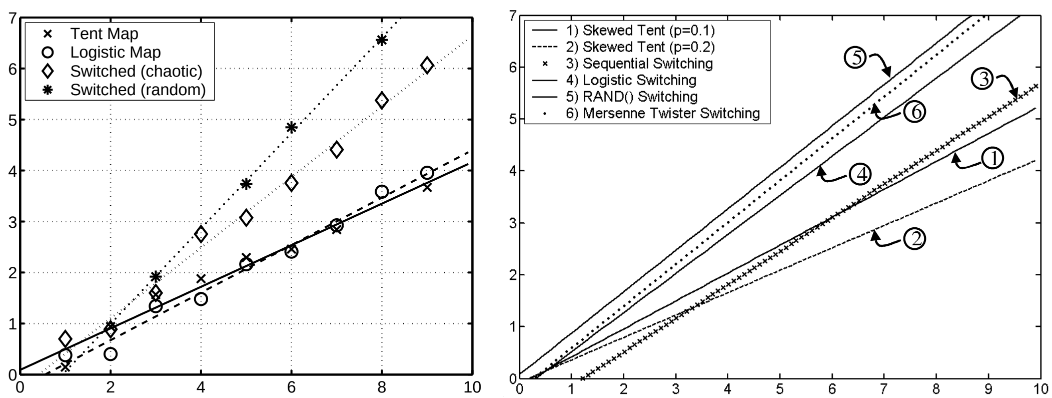

Fig. 2. Effect of switching on average period length: (a) Left: Chaotic switching yields $T = 10^{-0.2824}\varepsilon^{-0.6924}$ and Random switching using RAND(.) yields $T = 10^{-0.8766}\varepsilon^{-0.9351}$. Sequential switching (not shown in graph) yields $T = 10^{0.0137}\varepsilon^{-0.4270}$. (b) Right: Similar results were obtained for switching between skewed Tent maps (p=0.1 and 0.2). Observe that in both cases, T follows the following order: No switching < Sequential switching < Chaotic switching < Random switching.

It can be inferred from Figure 2 that switching increases the average period length as indicated by the increase in the slope of the linear fit.

3 Pseudo-random number generator using switching

It is well known that chaotic dynamical systems exhibit unpredictability, ergodicity and mixing properties. This suggests that chaotic maps can be used in generating pseudo-random numbers. Pseudo-random numbers are those which are "random-like" in their statistical properties. In 1947, Ulam and von Neumann suggested using the Logistic map to generate a sequence of pseudo-random numbers. Pseudo-random numbers are used in a variety of applications such as in Monte-Carlo simulations for random sampling from a distribution and are central in cryptographic applications to build stream and block ciphers and in several protocols requiring generation of random data. Pseudo-random number generators are algorithms implemented on digital systems that can generate these numbers. Due to limitations in computation and precision, pseudo-random number sequences are necessarily periodic (as opposed to an ideal random number generator which is a discrete memoryless information source that generates equiprobable non-periodic symbols; we shall not discuss these here). Sequences generated by pseudo-random number generators are expected to have large periods and pass a number of statistical randomness tests. In this paper, the phrase *random numbers* refers to uniformly distributed pseudo-random numbers.

The relationship between chaos and cryptography has been discussed by Kocarev [6]. Various one-dimensional chaotic maps have been proposed for generating random numbers, e.g: PL1D [7], LOGMAP [8] etc. In their study of the Logistic Map, Pathak and Rao [8] propose a pseudo-random number generator which has a period of about 10^8 when implemented in double precision. This period is quite small when compared to many other 'good' random number generators in the literature. They conjecture that such a period is due to the fact that the

value of a in the Logistic map: $ax(1-x)$, becomes slightly less than 4 because of which the map settles down into windows (attracting periodic orbits, see Figure 3(a)). We define the absence of attracting periodic orbits (for a particular value of the bifurcation parameter) as 'Full Chaos'. For the Logistic map, $a = 4$ exhibits 'Full Chaos'. The fact that Full Chaos exists only for a small set of parameters is a major hindrance in using chaotic maps as pseudo-random number generators. The limitations of computation and precision cause the parameters to deviate from Full Chaos values and this may result in settling on an attracting periodic orbit. Another major disadvantage of using chaotic maps directly is that the the successive points are strongly correlated. This shows up in the 2-dimensional phase space plot of the iterates.

To overcome the problem of low period lengths when using chaotic maps on finite precision computer, one of the strategies might be to switch between different chaotic maps and use them as a sequence of random numbers. We have already shown in Section 2 that switching between chaotic maps increases the average period length. We have also found that switching between more number of maps increases the average period length further (we have not indicated these results for want of space). However, we still need to evolve a strategy to avoid settling into windows or attracting periodic orbits (only under infinite precision, in finite precision it is still an open problem whether this has any benefits or not). To this end, we suggest using maps with the special property that they exhibit Full Chaos for a neighborhood of the parameter space unlike the logistic map which exhibits Full Chaos only at a single point ($a = 4$). This special property is defined as Robust Chaos. Chaos which fails to satisfy this special property is termed as Fragile Chaos (for eg., Logistic map, Tent map and most well known maps exhibit Fragile Chaos).

3.1 Robust Chaos maps

Robust Chaos is defined by the absence of periodic windows and coexisting attractors in some neighborhood of the parameter space [9]. Barreto [10] had conjectured that robust chaos may not be possible in smooth unimodal one-dimensional maps. This was shown to be false with counter-examples by Andrecut [11] and Banerjee [9]. Banerjee demonstrates the use of robust chaos in a practical example in electrical engineering. Andrecut provides a general procedure for generating robust chaos in smooth unimodal maps [12].

As observed by Andrecut [11], robust chaos implies a kind of ergodicity or good mixing properties of the map. This makes it very beneficial for cryptographic purposes. The absence of windows would mean that the these maps can be used in hardware (analog) implementation as there would be no *fragility* of chaos with noise (for eg., thermal noise) induced variation of the parameters (it would be impossible in practice to maintain a constant value, $a = 4$, in any analog implementation of Logistic Map). As it is impossible to eliminate noise in any hardware (analog) implementation, this property of Robust Chaos would be beneficial.

3.1.1 Some maps where we encountered Robust Chaos

A list of maps that we found to exhibit Robust Chaos are given below. Here $x \in [0, 1]$.

1. $x \mapsto \beta x - \lfloor \beta x \rfloor$ where β is any positive real number ($\neq 1$).
2. $x \mapsto \frac{x}{p}$ if $0 \leq x \leq p$ and $x \mapsto \frac{1-x}{1-p}$ if $p < x \leq 1$. The well known Tent map belongs to this family of maps ($p = 0.5$).
3. Two parameter Robust Chaos map [13]: $Skew - nGLS(a, p, x) = \frac{(a-p)+\sqrt{(p-a)^2+4ax}}{2a}, 0 \leq x < p; = \frac{(1+a-p)-\sqrt{(p-a-1)^2+4a(1-x)}}{2a}, \quad p \leq x \leq 1, \quad 0 < a \leq min(p, 1-p)$.
4. Andrecut and Ali [12] provide a novel method of converting any chaotic (not robust) 1D unimodal map $\phi(x)$ (a map that is $\mathcal{C}^3$ on [0,1] and which contains a single unique critical point 'c', actually a maximum) which has negative Schwarzian derivative to another unimodal map $f_\nu^{(\pm)}(x)$ that exhibits robust chaos given by:

$$f_\nu^{(\pm)}(x) = \frac{1 - \nu^{\pm\phi(x)}}{1 - \nu^{\pm\phi(c)}}, \quad \forall \nu > 0, \nu \neq 1. \tag{1}$$

As an example, the following maps, both derived from the unimodal map $\phi(x) = x(1-x)$ generate robust chaos:

$$f_\nu^+(x) = \frac{1-\nu^{+x(1-x)}}{1-\nu^{+0.25}}, \nu \in (0,1). \quad f_\nu^-(x) = \frac{1-\nu^{-x(1-x)}}{1-\nu^{-0.25}}, \nu \in (1,\infty). \tag{2}$$

5. The B-exponential map [14] which is a generalization of the Logistic map exhibits robust chaos. This will be discussed in 3.2.
6. One can use the fact that topological conjugacy preserves dynamics to generate a number of family of maps that all exhibit robust chaos. As an example, by applying the diffeomorphism $C(x) = \frac{(1-cos\pi x)}{2}$ to the B-exponential map, we can easily obtain a generalization of the Tent map that exhibits robust chaos [14].

3.2 B-Exponential map $GL(B,x)$

The B-Exponential Map $GL(B,x)$ is defined as follows:

$$GL(B,x) = \frac{B - xB^x - (1-x)B^{1-x}}{B-\sqrt{B}}, \qquad 0 \le x \le 1 \text{ and } B \in \mathbb{R}^+, B \neq 1. \tag{3}$$

Here, B is the bifurcation parameter. Note that $x_{n+1} = GL(B,x_n)$ is the iteration function.

3.2.1 Properties of B-exponential map

1. $GL(B,x)$ is unimodal for $e^{-4} \le B < \infty$ (unimodal implies that the map has only one critical point in [0,1] and it passes through zero at 0 and 1).
2. The B-Exponential Map is a generalization of the Logistic map because of the following interesting property:
$$\lim_{B\to 1} GL(B,x) = 4x(1-x). \tag{4}$$
This can be derived by a simple application of L'Hospital's rule. This property is the reason behind the notation '$GL(B,x)$' where 'GL' stands for Generalized Logistic. The maps of $GL(B,x)$ looks similar to the Logistic map for values of B near 1. It is interesting to note that $GL(B,x)$ tends to a constant function (with value 1) as B tends to ∞ (for all x).
3. The Lyapunov exponent of $GL(B,x)$ is a constant for all $B \ge e^{-4}$ and equals $\ln 2$. Thus the B-Exponential map is chaotic for all real $B \ge e^{-4}$. See [14] for details.
4. The B-Exponential Map exhibits *Robust Chaos* for $B \ge e^{-4}$. See [14] for details.

3.3 BEACH

We propose a pseudo-random number generator based on B-Exponential Map with the name BEACH (**B**-**E**xponential **A**ll-**C**haotic map-switc**H**ing). As the name suggests, the pseudo-random number generator is based on the principle of switching from map to map to extract numbers for the generator. Such a scheme has been studied by Rowlands [15] and Zhang [16]. Their methods were limited by the choice of maps and the kind of switching (or hopping as they call it) mechanism. MMOHOCC of Zhang [16] uses a finite number of arbitrarily predefined chaotic maps. They use pre-defined switching patterns to extract points from the trajectories. We propose a different switching mechanism, one that is chaotic. We also have the advantage of choosing from a very large number of Robust Chaos maps. Zhang's MMOHOCC has the problem of not having Robust Chaos for any of their maps. The maps they use (Chebyshev and Logistic) do not exhibit chaos for all values of the parameter. They use the fully chaotic value of $a = 4$ for the Logistic Map. However, such a method would have the draw-back of not being

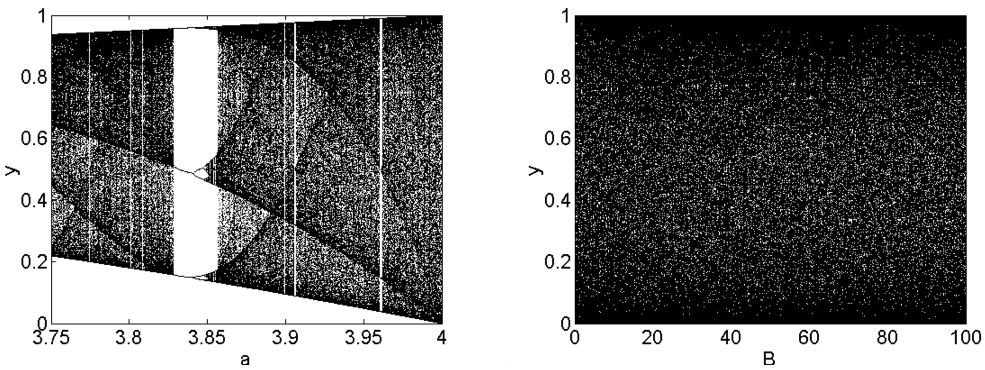

Fig. 3. (a) Left: A portion of the bifurcation diagram for the Logistic family ($x \mapsto ax(1-x)$) showing attracting periodic orbits (windows). This is termed as *Fragile* Chaos. (b) Right: Bifurcation diagram for the B-exponential map showing no windows. This is termed as *Robust* Chaos. This property is useful in generating pseudo-random numbers as we shall demonstrate.

fully chaotic when implemented in hardware. It is impossible to maintain a constant value for a parameter exactly in hardware owing to noise. Another problem they have to worry about is the presence of *attracting* periodic orbits for some values of the parameters even in the chaotic regime (this is indicated by the windows in the bifurcation diagrams). Our method eliminates all these problems. Although our method of using Robust Chaos maps would also settle down to periodic orbits in finite precision, this would not be case in an analog implementation. It is still not known what *repelling* periodic orbits would translate to in finite precision and whether this provides any real benefit in pseudo-random number generators.

3.3.1 The Algorithm

Figure 4 shows the flow chart for BEACH. $\{Z_i\}$ is the output sequence of pseudo-random numbers of length N. There are two seeds X_0 and B_0 to the algorithm (numbers between 0 and 1). X_0 forms the initial value of the iteration. We assume that these seeds are generated using a *random* procedure like the movement of the mouse, the speed of typing on the keyboard or some physical characteristic (like heat dissipation) in the hardware. In BEACH, each random number is picked from a particular map. The maps are generated parametrically using a sequence of B's, $\{B_1, B_2, ..., B_M\}$ where M is the number of maps we wish to use for switching. In our implementation, we pick one iterate from each of the B-exponential maps ($GL(B, x)$). Thus $M = N$, the length of the pseudo-random number sequence $\{Z_i\}$ we intend to generate (however the B's are not necessarily distinct owing to computer precision, though in theory there are an infinite number of them).

This sequence of B's can be generated in many ways. We only need to ensure that successive B's are not sufficiently close with a high frequency so that any two consecutive maps differ considerably. One way of varying B is by using the Logistic Map (we take 1/value of the Logistic Map). Alternatively, B can also be varied using the standard Tent map. Such a scheme ensures that successive B's are not close to each other on the real line for most of the times. We could also vary B using the orbit of BEACH itself. For our implementation, we use the Logistic Map for generating B's. Although varying B according to the Logistic Map does not give a random sequence of B's, it is sufficient for the purpose of switching maps. Each of the B-exponential maps are periodic because of finite precision, but as we showed in Section 2, by switching between maps, the average period length increases considerably.

We limit the value of B to 10,000 because the maps tend to the unit function for very large B. This may result in small period lengths or fixed points owing to limitations of precision on a computer (values very near to 1 may be rounded off to 1 which becomes 0 in the next iteration). If an iterate of the Logistic Map is lesser than 10^{-4}, it is replaced by an iterate of

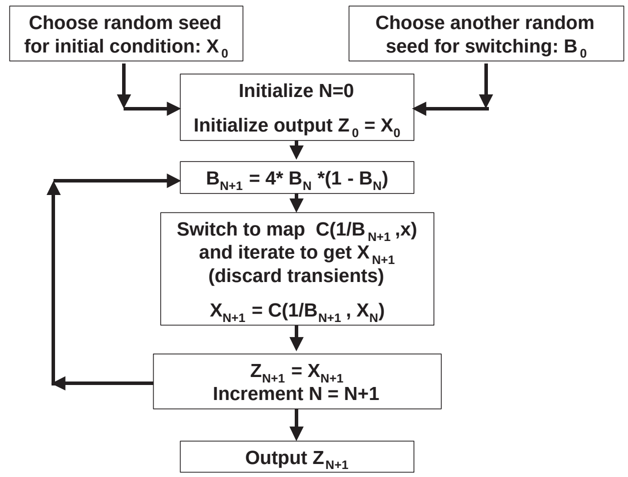

Fig. 4. Flow chart for BEACH pseudo-random number generator. Here, $C(B, x)$ is any Robust Chaos family of maps. For our implementation, we have chosen $C = GL(B, x)$.

the B-Exponential Map. If the iterate of B-Exponential Map is also less than 10^{-4}, then the Logistic Map iterate is set to 10^{-4}. Thus, we ensure that B does not exceed $10,000$. The final obtained value of Z_N is between 0 and 1 in double precision. To convert this to an integer, we multiply the iterate by 2^{52} (similar to Zhang's method [16]).

We also ensure that we do not use 0.75 as the seed for B_0 since it is the fixed point of the Logistic Map. The other disallowed seeds are 0 and 1 for obvious reasons.

The implementation of the algorithm was written in ANSI C in double precision floating point arithmetic. It is very hard to analytically determine the period of BEACH. Theoretically, robust chaos implies that there are no stable periodic orbits and we also know that the measure of periodic orbits is zero (in Full Chaos). However, when implemented on a computer, all orbits are periodic owing to limited precision. Since we have implemented BEACH in double precision arithmetic, the number of chaotic maps available for switching is around 10^{300} which would imply a substantial increase in average period length. As we are switching in a chaotic fashion, consecutive maps from which random numbers are extracted will be considerably different.

4 Randomness Evaluation of BEACH pseudo-random number generator

Figure 5(a) shows the histogram of BEACH output which appears uniform. The 2D phase space plots of BEACH output shown in Figure 5(b) appears random. To statistically confirm this, we tested using 3 standard test suites – The National Institute of Standards in Technology's Statistical Test Suite (NIST) [17,18], George Marsaglia's Diehard Battery of tests [19], and the ENT test [20]. These tests are well known in the cryptography community and are routinely used for evaluation of random number generators. The BEACH pseudo-random number generator successfully passed all the tests.

4.1 Entropy, Chi-square and Mean

Shannon's entropy is defined as $H(X) = -\sum_x P(x) \log_2 P(x)$ whenever $P(x) \neq 0$, where $P(x)$ is the probability that the random variable X is in the state x. Shannon's entropy is a measure of the *information density* of the data and a good measure of the degree of disorder (randomness) in the data. We created several binary files with the random numbers (taken as 32 bit integers) from BEACH (0 and 1 are the two states). An optimal compression using the ENT Pseudo-random Number Sequence Test Program (by John Walker) [20], resulted in an entropy of 1.0 bits per symbol, consistently for all the files. Thus, the program was unable to compress the

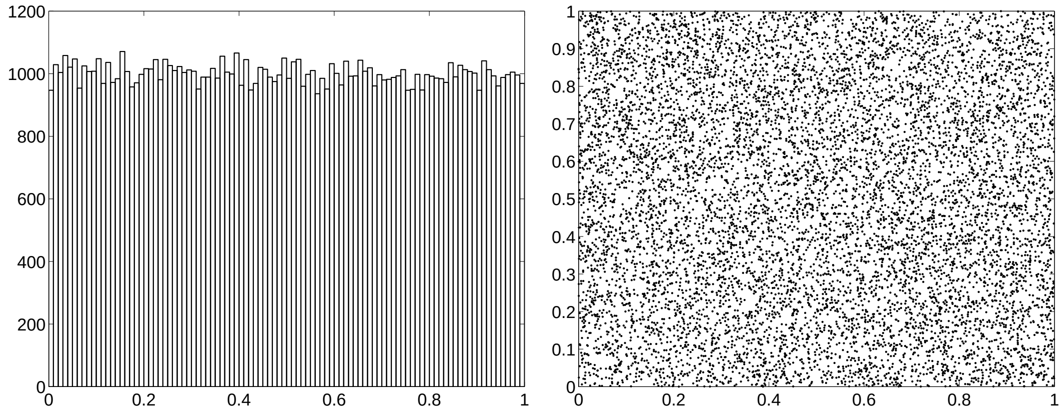

Fig. 5. (a) Left: The histogram of 10^5 pseudo-random numbers of BEACH. (b) Right: 2D phase space plot of 10^4 pseudo-random values of BEACH.

file. This is a strong evidence that BEACH is a good pseudo-random number generator. This is supported by the fact that the file also passed the Lempel-Ziv Compression test which is a part of the NIST Statistical Testing Suite.

The chi-square test is a very basic test of randomness. Knuth [21] gives a detailed treatment of the chi-square test. The chi-square distribution is computed for a sequence file and expressed as an absolute number and a percentage which indicates how frequently a truly random sequence would exceed the value calculated. This percentage is a measure of the randomness. If the percentage is less than 1% or greater than 99%, then the sequence is not random. Percentages between 90% and 95% and 5% and 10% indicate the sequence is "almost suspect" to be non-random [21]. Sequences generated by BEACH were within 25% to 75% consistently.

The mean of 1 billion bit sequences of BEACH was consistently at 0.5 for 1 bit word length and 127.5 for 8 bit word length. This is reported as part of the ENT test (Table 1 in Appendix). The serial correlation was also very low, of the order of 10^{-6} for a billion bit sequence. In addition to this, ENT program carried out Monte Carlo Value of Pi test. Each successive sequence of 24 bits are used as X and Y co-ordinates within a square. If the distance of the randomly-generated point is less than the radius of the circle inscribed within the square, the 24-bit sequence is considered a *hit*. The percentage of hits is used to calculate the value of π. For very large streams (this approximation converges very slowly), the value will approach the correct value of π if the sequence is close to random. For BEACH, the error percentage was consistently 0.0% (statistically). For the complete ENT test results, visit $http://mahesh.shastry.googlepages.com/entres/$. Table 1 in Appendix lists the value of these parameters for different lengths of generated random bits.

4.2 Other well known statistical test suites

BEACH random numbers also passed the NIST Statistical Test Suite [17] which consists of 15 tests. *Passing* of a test in NIST Suite implies a confidence level of 99%. In other words, when the p-value is more than the passing level, the test is considered passed with a confidence level of 99%. The details of the 15 tests in the NIST suite and their interpretation can be found in [17].

The Diehard Battery of Tests of George Marsaglia [19] are collectively considered to be one of the most stringent statistical tests for randomness. Ten streams of 1 billion bits each were generated using ten different random seeds. Each of the seed was chosen randomly from 10 equally spaced intervals from the set $(0, 1)$. The criteria for passing a Diehard test is that the p-value should not be 0 or 1 up to 6 decimal places. BEACH passed all the tests recommended in

the Diehard Battery. The test results are tabulated in Table 2 in the Appendix. The full results of the Diehard Tests are available at *http://mahesh.shastry.googlepages.com/diehard/*.

Furthermore, we found that BEACH successfully passes all the tests for extremely large sequences (we tested up to 10 Gb). In general, it is true that passing of these stringent tests only means a failure to falsify that the sequence is random. It does not mean that the sequence is actually random. However, since we have shown empirically that switching of chaotic maps does considerably increase the average period length, this may be the reason for the success of BEACH. An open problem is the determination of the exact relationship (on the lines of those established by Grebogi *et.al.*) between the average period length, computer precision, correlation dimensions of the maps, the number of maps being switched and the type of switching. Such a relationship will help us determine a bound on the average period length of BEACH pseudo-random number generator.

5 Conclusions

We have investigated the effect of different switching strategies on the average period length of simple chaotic maps (Tent map, Logistic Map and Skewed Tent maps) when implemented in finite precision. We have found that the average period lengths are in the following order (from smaller to bigger): No switching < Sequential switching < Chaotic switching < Random switching. Furthermore, the average period length increases with the number of maps being switched (we did not report the actual graphs in this case for want of space). We then introduced a generalization of the Logistic map which exhibits Robust Chaos and developed a pseudo-random number generator using switching of Robust Chaos maps for the first time. Robust Chaos which exhibits no attracting periodic orbits seems to be desirable for cryptographic applications, especially for hardware (analog) implementations since noise induced variations of the parameter does not result in windows. We have shown that our proposed pseudo-random number generator successfully passes stringent statistical tests of randomness which are well accepted in the cryptographic community.

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References

1. C Grebogi, E Ott, A Yorke, Phys. Rev. A **38**, (1988) 3688
2. P Grassberger, I. Procaccia, Phys. Rev. Lett. **50**, (1983) 346
3. S Wang, W Liu, H Lu, J Kuang, G Hu, arXiv:nlin.CD/0309005 v2 (2004)
4. S Li, G Chen, X Mou, Intl. Journal of Bifurcation and Chaos, **15**, (2005) 3119
5. M Matsumoto, T Nishimura, ACM Trans. Model. Comput. Simul. **8**, (1998)
6. L Kocarev, IEEE Circuits and Sys. Mag., (2001)
7. T Stojanovski, L Kocarev, IEEE Trans. on Circuits and Sys.-I: Fundamental Theory and Appl. **48**, (2001)
8. S C Pathak, S Suresh Rao, Phy. Rev. E **51**, (1995) 3670
9. S Banerjee, J A Yorke, C Grebogi, Phy. Rev. Lett. **80**, (1998) 3049
10. E Barreto, B R Hunt, C Grebogi, J A Yorke, Phy. Rev. Lett. **78**, (1997) 4561
11. M Andrecut, M K Ali, Europhys. Lett. **54**, (2001) 300
12. M Andrecut, M K Ali, Phy. Rev. E **64**, (2001) 025203

13. N Nagaraj, P G Vaidya, K G Bhat, Comm. in Nonlinear Sci. and Numerical Sim. **14**, 1013 (2009)
14. M C Shastry, N Nagaraj, P G Vaidya, arXiv.org:cs/0607069 (2006)
15. T Rowlands, D Rowlands, Intl. Conf. on Information Tech., (2002) 1
16. X Zhang, L Shu, K Tang, arXiv.org:cs/0601010 (2006)
17. National Institute of Standards and Technology, *Random Number Generation and Testing*, http://csrc.nist.gov/rng
18. A Rukhin *et. al.*,*A Statistical Test Suite for Random and Pseudorandom Number Generators for Cryptographic Applications*, NIST Special Publication 800-22, (2001)
19. G Marsaglia, *The Diehard Battery of Tests of Randomness*, http://www.csis.hku.hk/ diehard/index.html
20. John Walker, *ENT: A Pseudorandom Number Sequence Test Program*, http://www.fourmilab.ch/random/
21. D Knuth, *The Art of Computer Programming, Volume 2: Seminumerical Algorithms, Third Edition* (Reading, Massachusetts: Addison-Wesley, 1997)

Appendix

Table 1. Results of ENT on 3 bitstreams generated by BEACH.

Length	Entropy (per bit)	Chi-square distribution(%)	Arithmetic mean	Monte Carlo value of π (error %)	Serial correlation coeff
100 Mb	1.000000	50.00	0.5000	0.01	0.000151
500 Mb	1.000000	50.00	0.5000	0.00	0.000024
1 Gb	1.000000	75.00	0.5000	0.01	0.000035

Table 2. Results (p-values of hypothesis testing) of Diehard battery of tests on 10 bitstreams generated by BEACH, each of length 1 Gb. The criteria for passing a Diehard test is that the p-value should not be 0 or 1 up to 6 decimal places.

Test No.	Seed 1	Seed 2	Seed 3	Seed 4	Seed 5	Seed 6	Seed 7	Seed 8	Seed 9	Seed 10
1	.871	.221	.250	.922	.014	.154	.069	.050	.790	.515
2	.971	.527	.701	.273	.448	.946	.292	.460	.902	.113
3	.761	.942	.321	.679	.407	.585	.519	.587	.448	.962
4	.374	.801	.193	.098	.799	.117	.651	.437	.105	.015
5	.419	.665	.317	.324	.040	.876	.678	.507	.468	.075
6	.925	.199	.869	.401	.978	.998	.427	.930	.901	.268
7	.005	.192	.935	.611	.505	.621	.729	.339	.125	.773
8	.516	.549	.634	.539	.092	.483	.842	.053	.171	.576
9	.445	.542	.080	.964	.724	.773	.807	.136	.383	.806
10	.244	.084	.982	.779	.355	.088	.678	.324	.185	.059
11	.717	.440	.045	.269	.280	.376	.542	.873	.589	.952
12	.566	.702	.008	.900	.145	.065	.427	.784	.292	.065
13	.544	.636	.096	.376	.768	.922	.324	.903	.987	.677
14	.804	.835	.845	.302	.390	.791	.235	.567	.802	.746
15	.082	.512	.523	.875	.713	.604	.704	.909	.482	.119

The names of the tests (1-15) are: Birthday Spacings Test, Overlapping 5-Permutation Test, Binary Rank Test for 31×31 Matrices and 32×32 Matrices, Binary Rank Test for 6×8 Matrices, Bitstream Test, Tests OPSO, OQSO and DNA, Count-The-1's Test On A Stream Of Bytes, Count-The-1's Test for Specific Bytes, Parking Lot Test, Minimum Distance Test, 3Dspheres Test, Squeeze Test, Overlapping Sums Test, Runs Test, Craps Test.